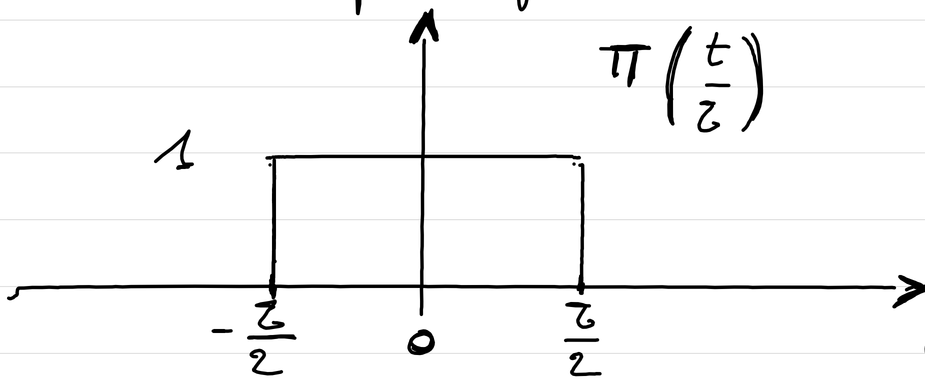
Fourier Transform of a Box Car



$$\Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \text{for } |t| \leq \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}.$$

Let's call $x(t) = \Pi\left(\frac{t}{\tau}\right)$ and

$\mathcal{F}[x(t)] = X(\omega)$. (Fourier Transform)

So $X(\omega) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-i\omega t}\,dt = -\frac{1}{i\omega}\left[e^{-i\omega t}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$

$$X(\omega) = \frac{1}{i\omega}\left[e^{i\omega\frac{\tau}{2}} - e^{-i\omega\frac{\tau}{2}}\right].$$

In order to introduce $\omega\frac{\tau}{2}$ we multiply by $\frac{2\tau}{2\tau}$.

Then $X(\omega) = \dfrac{\tau}{\omega\frac{\tau}{2}}\underbrace{\left(\dfrac{e^{i\omega\frac{\tau}{2}} - e^{-i\omega\frac{\tau}{2}}}{2i}\right)}_{\sin\left(\omega\frac{\tau}{2}\right)}$,

see Euler's formula

and finally $X(\omega) = \tau\left(\dfrac{\sin\left(\omega\frac{\tau}{2}\right)}{\omega\frac{\tau}{2}}\right)$

This function is called "Sine Cardinal"

In Mathematics $\operatorname{sinc}(x) = \dfrac{\sin x}{x}$,

whereas in signal processing

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}.$$

normalized

Both defined for $x \neq 0$

$$\Pi\left(\frac{t}{\tau}\right) \xrightarrow{FT} \tau \operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$$

The Fourier Transform of a Box Car function is a sine cardinal.