Filter (signal processing) with `python`

1. Using the python functions at your disposal in the `synthetic.py` toolbox[1], create a synthetic timeseries spanning the [1850.0 AD-2012.0 AD] time window (with a sampling rate of 12 points per year). The signal relies on five harmonic components of periods

$$\begin{aligned} T_1 &= 0.75 \text{ yr} \\ T_2 &= 1.0 \text{ yr} \\ T_3 &= 7.5 \text{ yr} \\ T_4 &= 11.0 \text{ yr} \\ T_5 &= 60.0 \text{ yr} \end{aligned} \tag{1}$$

   For each period, the corresponding amplitude A_i and phase ϕ_i can be arbitrarily chosen, and

$$s(t) = \sum_{i=1}^{i=5} s_i(t) = \sum_{i=1}^{i=5} A_i \cos\left(\frac{2\pi t}{T_i} + \phi_i\right). \tag{2}$$

2. Plot $s(t)$ between 1850 AD and 2012 AD using `matplotlib.pyplot`. It is possible to start from the `example_plot.py` script and to modify it.

3. In order to perform a spectral analysis of $s(t)$, it's possible to start from previously written scripts or to modify the `example_spectrum.py` python script (which uses `fft.rfft` function). One can adapt the latter in order to plot the squared modulus of $\tilde{s}(f)$, where $\tilde{s}(f)$ denotes the Fourier transform of $s(t)$.

4. The `signal` processing package of scientific python provides the function for applying a Butterworth bandpass filter. Using `example_filtering.py` as guideline, extract the 11 yr component from $s(t)$. Plot the filtered signal against the original one, and compare as well the spectra.

5. What's happen when modifying the amplitudes of each component?

6. Why long period peaks are wider (in terms of frequency) than the peaks corresponding to $T = 0.75$ yr or $T = 1$ yr components?

7. What's happen when reducing/enlarging the filtered domain?

8. Explore the arguments of the `signal.butter` function.

9. Create a low-pass filter with a corner frequency of 0.01 yr^{-1} and overlay the corresponding reconstructed timeseries and spectrum in the existing plots.

10. Create a high-pass filter with a corner frequency of 0.3 yr^{-1} and overlay the corresponding reconstructed timeseries and spectrum in the existing plots.

11. By looking at the manual page of `scipy.signal.filtfilt`, make some suggestions in order to improve the sought harmonic component (11 yr in our case). One way is to artificially increase the length of the timeseries (using zero padding), which tends to reduce spurious side effects on the filtered signal. Why?

[1] This practical comes from Alexandre Fournier (fournier@ipgp.fr).