Nantes Université
Département des sciences de la Terre et de l'univers
http://www.univ-nantes.fr/sciences/dptstu

Notations

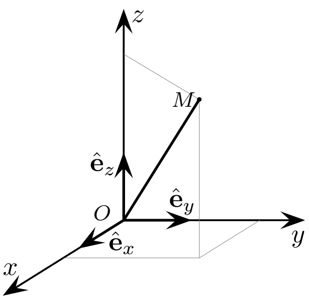
$$\mathbf{OM} = \overrightarrow{OM} \text{ (vecteur position)}$$

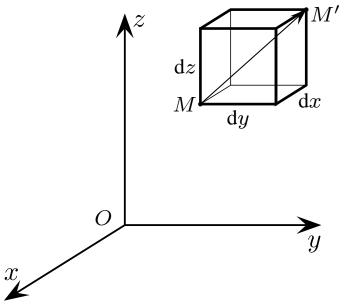
$$\mathbf{v} = \frac{\mathrm{d}\mathbf{OM}}{\mathrm{d}t} = \dot{\mathbf{OM}} \text{ (vecteur vitesse au point } M \text{ pour le déplacement } M \to M'\text{)}$$

$$\boldsymbol{\gamma} = \frac{\mathrm{d}^2\mathbf{OM}}{\mathrm{d}t^2} = \ddot{\mathbf{OM}} \text{ (vecteur accélération au point } M \text{ pour le déplacement } M \to M'\text{)}$$

$$\mathbf{dl} = \mathbf{MM}' = \overrightarrow{MM'} \text{ (vecteur déplacement élémentaire)}$$

1 Coordonnées cartésiennes



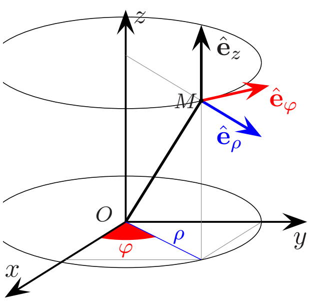


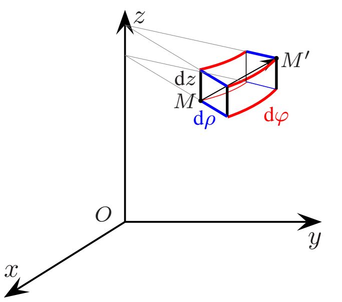
Le point M est repéré par les coordonnées (x, y, z), avec $-\infty < x < +\infty$, $-\infty < y < +\infty$ et $-\infty < z < +\infty$.

Trièdre $(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z)$ direct orthonormé
Facteurs d'échelle : $h_x = 1, h_y = 1, h_z = 1$
$\mathbf{OM} = x\,\hat{\mathbf{e}}_x + y\,\hat{\mathbf{e}}_y + z\,\hat{\mathbf{e}}_z$
$\mathbf{v} = \frac{\mathrm{d}x}{\mathrm{d}t}\hat{\mathbf{e}}_x + \frac{\mathrm{d}y}{\mathrm{d}t}\hat{\mathbf{e}}_y + \frac{\mathrm{d}z}{\mathrm{d}t}\hat{\mathbf{e}}_z = \dot{x}\,\hat{\mathbf{e}}_x + \dot{y}\,\hat{\mathbf{e}}_y + \dot{z}\,\hat{\mathbf{e}}_z$
$\boldsymbol{\gamma} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}\hat{\mathbf{e}}_x + \frac{\mathrm{d}^2y}{\mathrm{d}t^2}\hat{\mathbf{e}}_y + \frac{\mathrm{d}^2z}{\mathrm{d}t^2}\hat{\mathbf{e}}_z$
$\mathbf{dl} = \mathrm{d}x\,\hat{\mathbf{e}}_x + \mathrm{d}y\,\hat{\mathbf{e}}_y + \mathrm{d}z\,\hat{\mathbf{e}}_z$
$\mathbf{dS}_x = \mathrm{d}y\,\mathrm{d}z\,\hat{\mathbf{e}}_x$ (surface élémentaire pour $x = $ cste)
$\mathbf{dS}_y = \mathrm{d}x\,\mathrm{d}z\,\hat{\mathbf{e}}_y$ (surface élémentaire pour $y = $ cste)
$\mathbf{dS}_z = \mathrm{d}x\,\mathrm{d}y\,\hat{\mathbf{e}}_z$ (surface élémentaire pour $z = $ cste)
$\mathrm{d}\tau = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ (volume élémentaire)

$$\overrightarrow{grad}\, f = \boldsymbol{\nabla} f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}, \quad div\, \mathbf{V} = \boldsymbol{\nabla} \cdot \mathbf{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \quad \text{et} \quad \overrightarrow{rot}\, \mathbf{V} = \boldsymbol{\nabla} \times \mathbf{V} = \begin{pmatrix} \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \\ \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \\ \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \end{pmatrix}$$

2 Coordonnées cylindriques





Le point M est repéré par les coordonnées (ρ, φ, z), avec $0 \leq \rho < +\infty$, $0 \leq \varphi \leq 2\pi$ et $-\infty < z < +\infty$.

Trièdre $(\hat{\mathbf{e}}_\rho, \hat{\mathbf{e}}_\varphi, \hat{\mathbf{e}}_z)$ direct
Facteurs d'échelle : $h_\rho = 1, h_\varphi = \rho, h_z = 1$

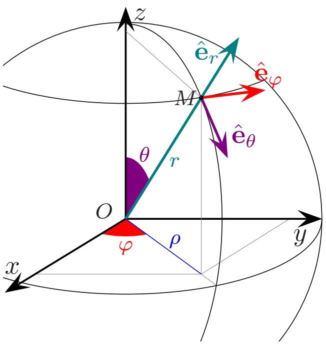
$$\begin{cases} \rho = \sqrt{x^2 + y^2} \\ \varphi = \arctan\left(\frac{y}{x}\right) \end{cases} \Leftrightarrow \begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{cases}$$

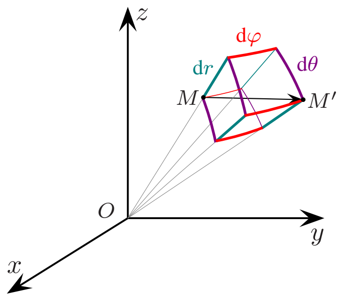
$\mathbf{OM} = \rho\,\hat{\mathbf{e}}_\rho + z\,\hat{\mathbf{e}}_z$
$\mathbf{v} = \dot{\rho}\,\hat{\mathbf{e}}_\rho + \rho\dot{\varphi}\,\hat{\mathbf{e}}_\varphi + \dot{z}\,\hat{\mathbf{e}}_z$
$\boldsymbol{\gamma} = (\ddot{\rho} - \rho\dot{\varphi}^2)\,\hat{\mathbf{e}}_\rho + (\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi})\,\hat{\mathbf{e}}_\varphi + \ddot{z}\,\hat{\mathbf{e}}_z$
$\mathbf{dl} = \mathrm{d}\rho\,\hat{\mathbf{e}}_\rho + \rho\,\mathrm{d}\varphi\,\hat{\mathbf{e}}_\varphi + \mathrm{d}z\,\hat{\mathbf{e}}_z$
$\mathrm{d}S_\rho = \rho\,\mathrm{d}\varphi\,\mathrm{d}z$ (surface élémentaire pour $\rho = $ cste)
$\mathrm{d}S_\varphi = \mathrm{d}\rho\,\mathrm{d}z$ (surface élémentaire pour $\varphi = $ cste)
$\mathrm{d}S_z = \rho\,\mathrm{d}\rho\,\mathrm{d}\varphi$ (surface élémentaire pour $z = $ cste)
$\mathrm{d}\tau = \rho\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z$ (volume élémentaire)

$$\boldsymbol{\nabla} = \frac{\partial}{\partial \rho}\,\hat{\mathbf{e}}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \varphi}\,\hat{\mathbf{e}}_\varphi + \frac{\partial}{\partial z}\,\hat{\mathbf{e}}_z$$

$$\boldsymbol{\nabla} f = \begin{pmatrix} \dfrac{\partial f}{\partial \rho} \\ \dfrac{1}{\rho}\dfrac{\partial f}{\partial \varphi} \\ \dfrac{\partial f}{\partial z} \end{pmatrix}, \quad \boldsymbol{\nabla}\cdot\mathbf{V} = \frac{1}{\rho}\frac{\partial\,(\rho V_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial V_\varphi}{\partial \varphi} + \frac{\partial V_z}{\partial z} \quad \text{et} \quad \boldsymbol{\nabla}\times\mathbf{V} = \begin{pmatrix} \dfrac{1}{\rho}\dfrac{\partial V_z}{\partial \varphi} - \dfrac{\partial V_\varphi}{\partial z} \\ \dfrac{\partial V_\rho}{\partial z} - \dfrac{\partial V_z}{\partial \rho} \\ \dfrac{1}{\rho}\left(\dfrac{\partial \rho V_\varphi}{\partial \rho} - \dfrac{\partial V_\rho}{\partial \varphi}\right) \end{pmatrix}$$

3 Coordonnées sphériques





Le point M est repéré par les coordonnées (r, θ, φ), avec $0 \leq r < +\infty$, $\quad 0 \leq \theta \leq \pi \quad$ et $\quad 0 \leq \varphi \leq 2\pi$.

Coordonnées géographiques

latitude : $\lambda = \pi/2 - \theta$, (θ est la colatitude)
longitude : $\phi = \varphi - \varphi_0$,
avec φ_0 le méridien de référence (Greenwich)

$(\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\varphi)$ trièdre direct
Facteurs d'échelle : $h_r = 1, h_\theta = r, h_\varphi = r\sin\theta = \rho$

$$\left\{\begin{array}{rcl} r &=& \sqrt{x^2+y^2+z^2} \\ \theta &=& \arccos\left(\frac{z}{r}\right) \\ \varphi &=& \arctan\left(\frac{y}{x}\right) \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} x &=& r\sin\theta\cos\varphi \\ y &=& r\sin\theta\sin\varphi \\ z &=& r\cos\theta \end{array}\right.$$

$\mathbf{OM} = r\,\hat{\mathbf{e}}_r$
$\mathbf{v} = \dot{r}\,\hat{\mathbf{e}}_r + r\dot{\theta}\,\hat{\mathbf{e}}_\theta + r\sin\theta\dot{\varphi}\,\hat{\mathbf{e}}_\varphi$

$$\begin{array}{rcl} \boldsymbol{\gamma} &=& (\ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta)\,\hat{\mathbf{e}}_r \\ && +(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\,\cos\theta)\,\hat{\mathbf{e}}_\theta \\ && +(r\ddot{\varphi}\sin\theta + 2\dot{r}\dot{\varphi}\sin\theta + 2r\dot{\theta}\dot{\varphi}\cos\theta)\,\hat{\mathbf{e}}_\varphi \end{array}$$

$\mathrm{d}\mathbf{l} = \mathrm{d}r\,\hat{\mathbf{e}}_r + r\mathrm{d}\theta\,\hat{\mathbf{e}}_\theta + r\sin\theta\mathrm{d}\varphi\,\hat{\mathbf{e}}_\varphi$
$\mathrm{d}\mathbf{S}_r = r^2\sin\theta\mathrm{d}\,\theta\,\mathrm{d}\varphi$ (surface élémentaire pour r = cste)
$\mathrm{d}\mathbf{S}_\theta = r\sin\theta\,\mathrm{d}r\,\mathrm{d}\varphi$ (surface élémentaire pour θ = cste)
$\mathrm{d}\mathbf{S}_\varphi = r\mathrm{d}r\,\mathrm{d}\theta$ (surface élémentaire pour φ = cste)
$\mathrm{d}\tau = r^2\sin\theta\,\mathrm{d}r\mathrm{d}\,\theta\,\mathrm{d}\varphi$ (volume élémentaire)

$$\boldsymbol{\nabla} = \frac{\partial}{\partial r}\,\hat{\mathbf{e}}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\,\hat{\mathbf{e}}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\,\hat{\mathbf{e}}_\varphi$$

$$\boldsymbol{\nabla} f = \begin{pmatrix} \dfrac{\partial f}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial f}{\partial \theta} \\ \dfrac{1}{r\sin\theta}\dfrac{\partial f}{\partial \varphi} \end{pmatrix}, \quad \boldsymbol{\nabla}\cdot\mathbf{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta V_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial V_\varphi}{\partial \varphi},$$

$$\boldsymbol{\nabla}\times\mathbf{V} = \frac{1}{r}\begin{pmatrix} \dfrac{1}{\sin\theta}\left(\dfrac{\partial\,(V_\varphi\sin\theta)}{\partial \theta} - \dfrac{\partial V_\theta}{\partial \varphi}\right) \\ \dfrac{1}{\sin\theta}\dfrac{\partial V_r}{\partial \varphi} - \dfrac{\partial\,(rV_\varphi)}{\partial r} \\ \dfrac{\partial\,(rV_\theta)}{\partial r} - \dfrac{\partial V_r}{\partial \theta} \end{pmatrix}.$$